University of Utah
UNDERGRADUATE RESEARCH JOURNAL

WEIGHING ABELL 2029: HOW DIFFERENT ASSUMPTIONS CHANGE A GALAXY CLUSTER'S MASS

Logan Thomas Kelley (Daniel Wik, PhD)
Department of Physics and Astronomy

ABSTRACT

At a given mass, the amount of galaxy clusters within some volume greatly depends on certain cosmological parameters. Examples of such parameters are: how much total mass there is (Ω_m), the equation of state parameter of dark energy (w), and the scale of fluctuations (σ_8). These quantities can be measured by finding the number of galaxy clusters as a function of mass. This would require finding accurate masses for these galaxy clusters, but finding such masses requires us to make underlying assumptions in order to make estimates and depends on how good the calibration was of the instruments that were used. By obtaining approximately 107 kiloseconds of Chandra observations of galaxy cluster Abell 2029, we can use the Chandra telescope's detection of Abell 2029's hot gas to derive its total enclosed mass. As mentioned above, deriving this total enclosed mass requires assumptions to be made. Such assumptions include assuming some kind of symmetry to simplify the problem, as well as assuming the equipment was calibrated correctly. By testing how the derived mass of Abell 2029 changes when we change our underlying assumptions, we can find out which assumptions dominate the systematic uncertainty of galaxy cluster mass measurements. Knowing which of these assumptions dominates the uncertainty helps guide us in what we need to do in order to better refine cosmological parameters.

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INTRODUCTION

For detailed explanations of various physics terms, see appendix [x].

In the field of cosmology, there exists numerous constants that surface in foundational equations such as the Friedmann equations, which govern the expansion of space in a homogenous (same everywhere) and isotropic (no preferred direction) model of the universe. Such constants are continuously being refined, including placing limits on them in order to increase the accuracy of said equations. They are also being refined in order to help achieve more correct measurements of various natural phenomena. An example of such natural phenomena is that of a galaxy cluster. Galaxy clusters are a collection of hundreds, if not thousands, of galaxies which are bound together by the force of gravity. In any given volume of the universe, the number of galaxy clusters (i.e. the abundance) as a function of mass heavily relies on certain cosmological constants. The constants most sensitive to the abundance of galaxy clusters include Ω_m, w, and σ_8. These constants can be measured through this function of galaxy cluster mass, but to accurately find the mass of a galaxy cluster requires certain underlying assumptions. Knowing which of these assumptions changes the resulting galaxy cluster mass the most would guide us in future efforts of refining these constants, and such knowledge is the end goal of this project.

For motivation, let us first briefly consider the named cosmological constants. The constant Ω_m is equal to the density of matter in the universe, and is part of another cosmological constant Ω_0, which is called the density parameter [1] and describes the energy density of everything in the universe. The density of matter has been estimated to be roughly 0.3, implying around 30% of the universe's density comes from matter. Refining this value for matter density would allow us to get a more accurate value for Ω_0, which in turn allows us to determine the fate of our universe. Additionally, a more accurate value of the density of matter would help us to better understand the makeup of our universe. With matter only being about 30% of the universe's energy density, where does the rest of the energy density come from? The answer to that is what is known as dark energy, the energy which is hypothesized to accelerate universal expansion.

The equation of state parameter of dark energy, w, is equal to the ratio between dark energy's pressure and energy density. Observations of this parameter have resulted in values very close to -1, and it is important to refine this value so that it is either equal to -1, or decisively different than this value. This comes from the fact that the classical explanation of dark energy as Einstein's cosmological constant has w = -1, and an opposing theory of dark energy, named "quintessence" [2], has $w \neq -1$. If w = -1, or more specifically if $w < -1/3$, then that would mean the expansion of the universe is accelerating, which has been observed. If $w < -1$, then the universe's ultimate fate would be a "big rip," or sometime in the future the expansion of space would be so strong it would tear matter and even spacetime apart. If $-1/3 > w > -1$ the expansion of the universe would still accelerate, but slower. If $w > -1/3$, there would be no accelerated expansion of the universe. Finally, we have σ_8 which is known as the scale of fluctuations. This refers to the strength of density variations in the early stages of the universe and therefore it is considered to be the seeds of all universal structure. As cosmology has developed rapidly since the turn of the century, these cosmological constants have been refined with relatively high precision, except for the scale of fluctuations which has been estimated to be anywhere between 0.7 and 1.1 (Primack, J. R. 2004). It is critical for cosmologists to place better limits on this parameter in order to better understand the early universe.

Within a galaxy cluster, there is a vast amount of extremely hot (10^7 kelvin or more) gas that makes up a majority of the cluster's mass. Since the gas is extremely hot, nearly all elements within are highly ionized. When a negatively charged electron comes close to one of these highly ionized atoms, it is deflected and loses some of its kinetic energy. This loss of kinetic energy produces a photon, which we can then observe. The energy of the emitted photon depends on the temperature of the electron, and for temperatures between 0.2 – 10 keV (or between 10^7 – 10^8 kelvin) the photon is in the X-ray range of the electromagnetic spectrum. This process is called thermal bremsstrahlung emission, and is the reason why galaxy clusters give off X-ray radiation. From this X-ray radiation we can derive the temperature and density of the cluster's gas. If we were to assume the cluster was in hydrostatic equilibrium, where the outward gas pressure is balanced by the inward gravity force, then we could estimate the mass of the galaxy cluster. Since we can relate the outward gas pressure to temperature through the ideal gas law, $P = NkT$, we can solve for the mass coming from the gravity equation to estimate the cluster's enclosed mass by only really needing the temperature and density of the gas. This makes getting a mass estimate rather easy, but it isn't always correct to assume hydrostatic equilibrium.

As previously stated, the three cosmological constants detailed in this paper can be measured by finding the abundance of galaxy clusters as a function of mass. A very accurate measure of the masses of these galaxy clusters would be required to obtain the desired limits on these constants. However, finding the masses of galaxy clusters requires certain underlying assumptions to be made and also depends on the calibration of the instruments used to get the data. By obtaining approximately 107 kiloseconds of observations of galaxy cluster Abell 2029 from the Chandra X-ray Observatory, we are able to use the Chandra telescope's detection of Abell 2029's hot gas to derive its total enclosed mass. An example of an assumption made during this derivation is assuming some kind of symmetry in order to simplify the problem. Often, galaxy clusters are assumed to be spherical and thus have spherical symmetry in order to make mass derivations easier. The symmetry of a galaxy cluster is dictated by dark matter. When dark matter clumps together gravitationally, some dark matter is left behind, creating filaments that connect galaxies to other astronomical objects. Galaxy clusters lie at the intersection of these filaments. Due to these filaments being of different sizes and densities, galaxy clusters are not always perfectly spherical and often do not have symmetry that is easy to work with. Another assumption is that the equipment being used was properly calibrated, which unfortunately is not always the case. Equipment which is not properly calibrated will cause the data to be modeled incorrectly, and thus would give incorrect values for mass. Assuming the gas within the galaxy cluster is in hydrostatic equilibrium also has room for error, as this is also not always the case. This assumption is constantly used in mass derivations as it allows cosmologists to use a relatively simple equation, the equation of hydrostatic equilibrium. Otherwise, the equations cosmologists would have to use would be much worse. Error can also come from assuming the regions you are extracting data from are properly centered. This appears to be an easy thing to check for, but it can be much harder than it looks, as it is the X-ray peak of the cluster that should be the center and not just the geometrical center. An additional assumption is having the correct dark matter halo mass function. For a cluster in hydrostatic equilibrium, this isn’t an assumption, but for other methods of mass estimation it is. Other uncertainties can come from assuming your value for redshift is correct, and from the lack of knowledge about galaxy cluster evolution and how certain variables scale/relate with each other.

By deriving the total enclosed mass of Abell 2029 with these assumptions, and then repeating the derivation by changing some of these assumptions, in addition to outside research found in physics papers, we are able to determine which of these assumptions dominate the systematic uncertainty of galaxy cluster mass estimates. Knowing which of these assumptions cause the biggest difference in the total enclosed mass estimate helps guide us in what we need to do in order to better refine these cosmological parameters. With our more accurate values for these parameters, we will hopefully be able to help answer some of the questions left unanswered by the ambiguity of the current values of the parameters, the most important of which being the question of the ultimate fate of our universe.

METHODS

To derive Abell 2029's mass, the first step was to reprocess the data obtained from Chandra. The specific Obs IDs and some of their properties can be seen in Table 1.

Observation ID	Exposure Time (kilosec)	Start Date
891	19.81	2000-04-12 06:37:52
4977	77.9	2004-01-08 12:57:46
6101	9.92	2004-12-17 03:33:41

Table 1 – The specific Obs IDs of Abell 2029 used in this project along with their respective exposure times and dates of observations.

All of the reprocessing was done in Python, and initially consisted of processing the level 1 event files from Chandra into level 2 event files. Level 2 event files are created from the level 1 event file by filtering on the good time intervals (GTI) and potential bad pixels. Following this, the light curve of Abell 2029 was checked for background flares and the level 2 event files were filtered to remove any high background periods. Then, the level 2 event files were filtered using the cleaned GTI and the readout background events were filtered to create level 3 event files. An appropriate ACIS blank sky background dataset was then produced for Abell 2029's dataset (where ACIS stands for Advanced CCD Imaging Spectrometer, which is the instrument on Chandra being used). This background dataset was necessary in order to subtract background from spectra. Abell 2029's spectra were extracted from an area of radius 3 arcminutes, centered on the cluster center. A spectrum was extracted from each CCD that had at least 10 counts in the 3-arcminute region, then these spectra were added together for each Obs ID, and finally the spectra from each Obs ID were added together. After this, a best fit single temperature model for the entire cluster was needed in order to produce the exposure maps. Through the use of XSPEC, an X-ray spectral fitting program, a single-temperature APEC model with simple absorption (apec*tbabs) was acquired. With this temperature model we were then able to create an exposure map, which measured the amount of time each of the telescope's CCDs was collecting photons as a function of energy. Having this exposure map allowed us to create an exposure corrected and background subtracted image of the cluster, which can be seen in Figure 1.

Figure 1 –The background subtracted exposure corrected image of Abell 2029.

Once the data were reprocessed, the next step was to obtain profiles for Abell 2029's temperature and density. This was achieved by finding the projected temperatures and surface brightness in concentric annular regions. These annular regions were chosen so that the total enclosed counts increased by the same amount for each larger circle. The smallest of the regions was 13 arcseconds in radius with approximately 44,000 counts. This number of counts was arbitrarily decided upon, and has no special meaning, but was chosen to be fairly large to help reduce the uncertainty in the temperature. Following regions increased by roughly 44,000 counts until the entire cluster was nearly covered, corresponding to 19 regions. Then, 5 additional regions were added in order to completely cover the cluster that increased by only 10,000 counts in order to help show the temperature fall off at about 1 Mpc from the cluster's center, where the emission is naturally fainter. Each specific annular region and their respective temperatures and errors can be seen in Table 2 below. These annular regions were then subject to the XSPEC temperature fitting mentioned earlier in order to obtain an array of best-fit temperature values for each region with their respective high and low errors. This array of temperature values, along with the images of the cluster that were produced earlier, allowed us to fit a surface brightness profile to Abell 2029. This profile was modeled using a two-β density model seen in equation 1, where r_1, r_2, β_1, β_2, n_1 and n_2 are best-fit constants.

$$n_e(r) = n_1\left(1+(r/r_1)^2\right)^{-3\beta_1/2} + n_2\left(1+(r/r_2)^2\right)^{-3\beta_2/2} \tag{1}$$

The best fit parameters obtained from this profile were then used in combination with the temperatures to obtain a temperature profile. This profile was modeled using the best fit model from Vikhlinin 2006 Eq. (4) seen in equation 2, where T_0, r_t, a, b and c are best-fit constants.

$$T(r) = \frac{T_0(r/r_t)^{-a}}{\left[1+(r/r_t)^b\right]^{c/b}} \tag{2}$$

Annulus Radii (inner-outer, arcsec)	Temperature (kT)	High Error (kT)	Low Error (kT)
0 - 13	4.99	0.05	0.07
13 - 21	6.33	0.14	0.13
21 - 28	6.61	0.14	0.11
28 - 35	7.47	0.21	0.21
35 - 43	7.31	0.16	0.21
43 - 51	6.97	0.22	0.20
51 - 60	7.63	0.16	0.15
60 - 70	7.60	0.21	0.22
70 - 81	8.19	0.21	0.23
81 - 94	8.50	0.32	0.21
94 - 108	8.36	0.23	0.22
108 - 124	8.21	0.17	0.22
124 - 142	8.74	0.19	0.19
142 - 163	8.49	0.44	0.24
163 - 189	8.41	0.28	0.23
189 - 221	8.35	0.28	0.26
221 - 258	8.10	0.28	0.29
258 - 333	8.36	0.42	0.30
333 - 544	8.07	0.23	0.23
544 - 614	6.08	0.47	0.44
614 - 701	5.18	0.43	0.35
701 - 797	3.99	0.24	0.24
797 - 914	3.01	0.18	0.23
914 - 1132	1.87	0.19	0.21

Table 2 – The sizes of each annular region in arcseconds, along with their best-fit temperature and respective high and low 1-sigma errors.

The best fit parameters from the temperature profile were then subsequently plugged back into the surface brightness profile in order to achieve a better fit, since emissivity depends on temperature. This emissivity-temperature relation can be found in Planck's law, or can be found reproduced in equation 3 where B represents the spectral radius.

$$\epsilon = 1.4 \times 10^{-27} T^{1/2} n_e n_p \bar{g}_{\mathrm{ff}}(T) \ \mathrm{erg/cm^3/s} \quad (3)$$

Thus, since emissivity and temperature are related, this process could be iterated until a satisfactory fit was achieved. Finally, once the surface brightness and temperature profiles were fit until each further iteration had a negligible effect on the parameters, their best fit parameters were used to obtain the hydrostatic mass of the galaxy cluster. The hydrostatic mass profile was fit using the model seen in equation 4, where k is the Boltzmann constant, g is the gravitational constant, m_p is the mass of a proton and μ is akin to the reduced mass of protons and electrons.

$$M(<r) = -\frac{kT(r)r}{G\mu m_p}\left[\frac{d \ln n_H}{d \ln r} + \frac{d \ln T}{d \ln r}\right] \quad (4)$$

For this equation, we plugged our n_e model in the place of the n_H, with the assumption that $n_e = n_H$. However, this value for the hydrostatic mass assumes the gas is in hydrostatic equilibrium, and that all of the pressure within the gas comes from thermal motion of particles. Computer simulations have shown that more than 10% of this pressure can come from non-thermal sources like turbulence and bulk motions. To correct for this, equation 16 in Pearce et al. 2019, reproduced in equation 5, was applied to Abell 2029's hydrostatic mass. The same equation the paper uses for the non-thermal pressure α was used here as well, and can be seen in equation 6.

$$M_{\mathrm{mod,corr}} = \frac{1}{1-\alpha}\left[M_{\mathrm{M,HSE}} - \frac{\alpha}{1-\alpha}\frac{k_{\mathrm{B}}Tr}{G\mu m_{\mathrm{p}}}\frac{\mathrm{d}\ln\alpha}{\mathrm{d}\ln r}\right] \quad (5)$$

$$\alpha(r) = 1 - A\left(1 + \exp\left[-\left(\frac{r/r_{500}}{B}\right)^{C}\right]\right) \tag{6}$$

In order to approximate the error in the hydrostatic mass, the T_0 parameter in equation 2 was raised and lowered until it had a chi squared value that differed from the best fit T_0 value by about 3. We chose to vary this T_0 value since the hydrostatic mass is directly proportional to it, and we also made the assumption that the density contribution to the error was negligible. Then, the error bars were created so that they included the masses found from raising and lowering this T_0 value.

During this process of determining the corrected hydrostatic mass, there were various miscenterings that were performed in order to test the uncertainty in the centering of the annular regions. There were 5 total miscenterings, with each roughly 9 arcseconds (or approximately 20kpc) away from the correct center, except for one which was roughly 26 arcseconds (or approximately 50 kpc) away in order to test a more extreme case. Then, the final uncertainty test that we performed was varying the n_H (hydrogen column density) value in the XSPEC temperature model to test the calibration. The n_H value was raised and lowered once with the values being picked so that the overall temperature was raised and lowered by roughly 10%.

RESULTS

3.1 – Nominal Mass Profile

The first result of this project was the hydrostatic mass of Abell 2029, without correcting for non-thermal pressure. As previously stated, this was achieved by iteratively fitting the temperature and surface brightness profiles until the iterations had minimal effects on the parameters. The best-fit surface brightness and model density can be seen in Figure 2 below, and the best fit temperature profile can be seen in Figure 3 below. Once that was done, the best-fit parameters that were found were used in conjunction with the hydrostatic mass profile (equation 4) to produce the hydrostatic mass of Abell 2029, seen in Figure 4 below. The total enclosed mass of Abell 2029 was calculated to be approximately $1.3x10^{15}$ solar masses. The above mentioned best fit parameters can be found in Tables 3 and 4 below. As previously stated, the error bars for the hydrostatic mass were roughly estimated by raising and lowering the T_0 value from equation 2 (since it is directly proportional to T(r)) until the corresponding chi squared value differed from the best fit value by about 3, which corresponds to roughly a 90% error. This resulted in about a difference of 0.8 keV for the lowered T_0 and 0.5 for the raised T_0. These T_0 values were then plugged into equation 3 to find their respective hydrostatic masses. The error bars were then created so that they incorporated these uncertainties, and the resulting error was found to be about 12%, which corresponds to the statistical uncertainty on the mass.

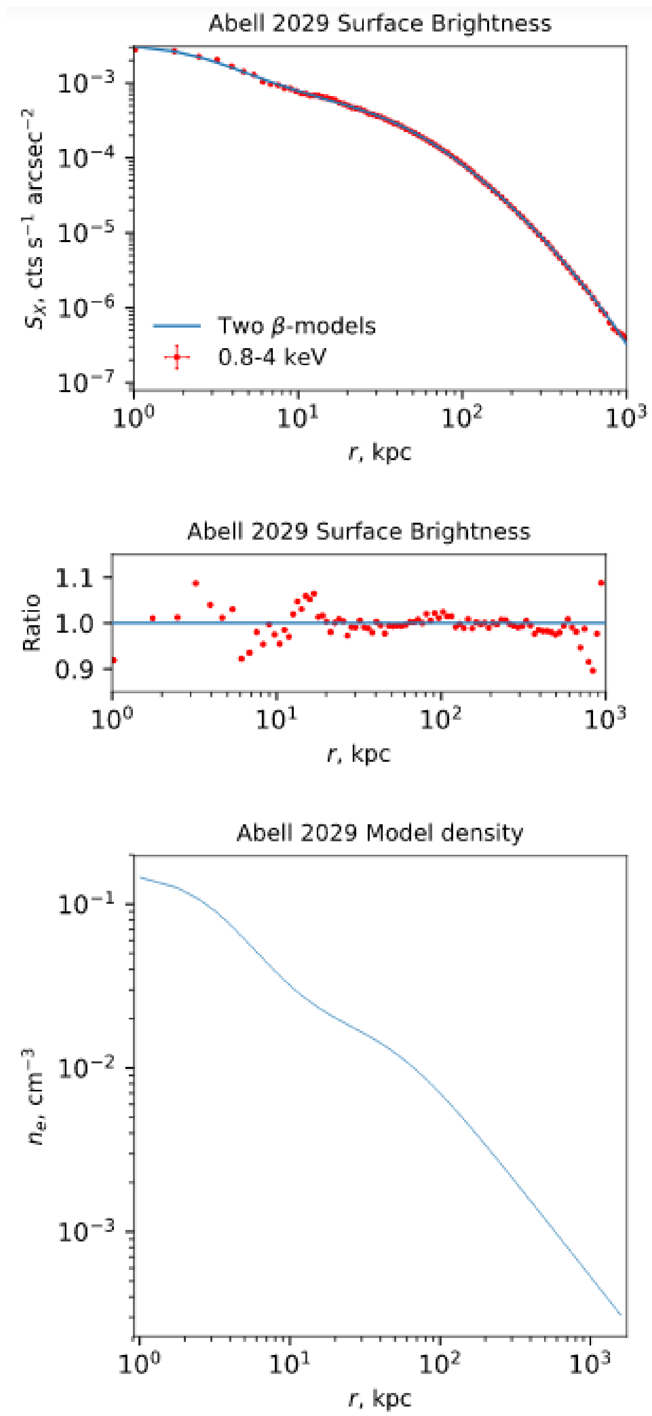

Figure 2 – The best-fit surface brightness and model density profiles for Abell 2029. In the top panel we have the surface brightness (measured in 0.8 – 9 keV) as a function of projected radius (shown by the red dots) compared to the best fit 2- β model (shown by the blue line). In the middle panel, we have the ratio between the surface brightness and the 2- β model to help illustrate the accuracy of the fit. In the bottom panel, we have the model density represented by the number of electrons per cubic centimeter.

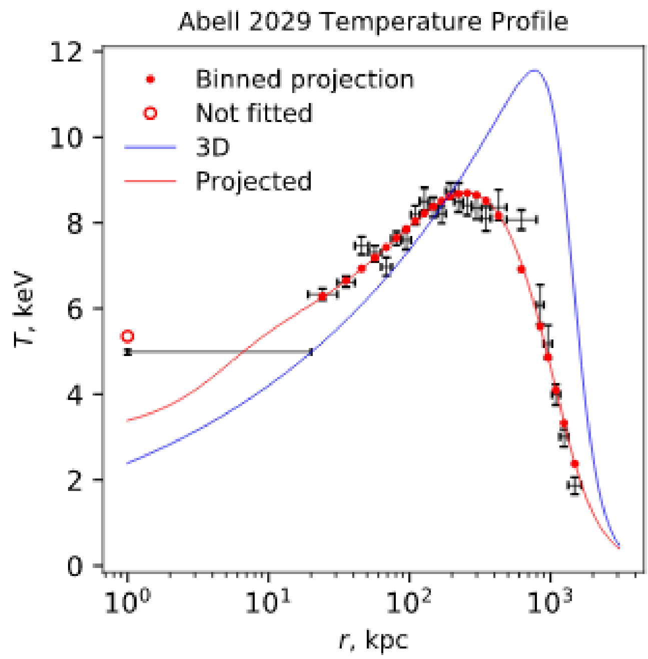

Figure 3 – Abell 2029's best-fit temperature profile. The red line represents the projected temperature of Abell 2029 in accordance with the apec*tbabs temperature model applied to each annular region. The blue line represents the 3D temperature profile of Abell 2029, which was calculated using the Vikhlinin 2006 temperature model, seen in equation 2.

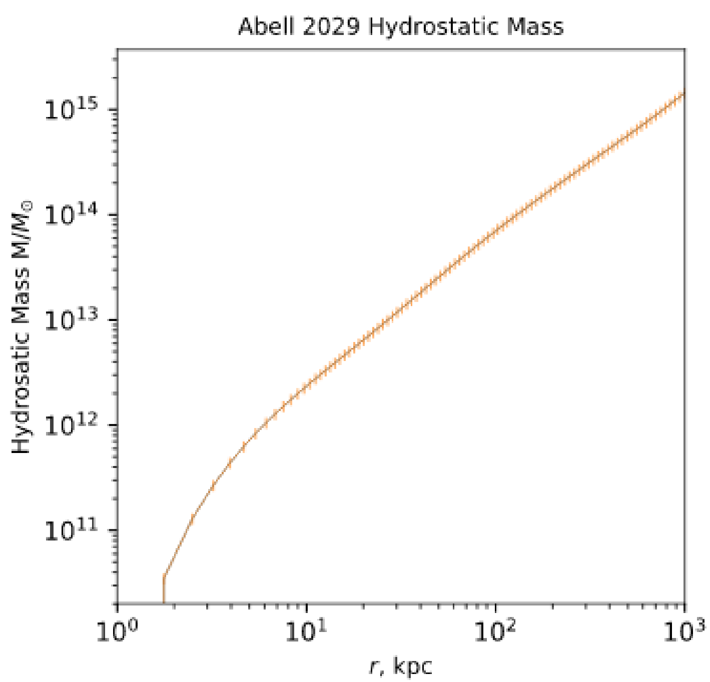

Figure 4 – Abell 2029 hydrostatic mass profile. The Hydrostatic mass at each radius is the total enclosed mass at that radius, M(< r).

β_1	β_2	r_1	r_2	n_1	n_2
0.39	0.63	49.93 kpc	3.21 kpc	1.7332e-02 cm^{-3}	1.3985e-01 cm^{-3}

Table 3 – The best fit parameters for Abell 2029's density profile (equation 1).

T_0	r_t	a	b	c	chi squared	N_{dof}
13.946 keV	1356.58 kpc	-0.24	5.00	4.33	30.594	19

Table 4 – The best fit parameters for Abell 2029's temperature profile (equation 2), as well as the chi squared value and the number of degrees of freedom (N_{dof}) for this fit.

3.2 – Region Centering

Once we had the hydrostatic mass of galaxy cluster with the appropriate assumptions to simplify the problem, the plan was to repeat this process to test for the impact of our assumptions on the final answer. The first thing we did was change the centering of the annuli from which we obtained the hydrostatic mass of the cluster. The centering of data gathering regions on galaxy clusters, or even normal galaxies, is something which takes much more skill to master than what it seems at surface level. Physicists cannot center these regions just in the exact middle of the galaxy cluster as this is not always the X-ray peak. The X-ray peak is what physicists look to center their regions on, unless this peak is substantially offset by certain effects of the ICM, of which a cold front would be an example. When there is evidence of a cold front or other kind of offset, the physicist will then want to center their regions on the center of the large-scale distribution of emission. Correctly centered regions are something which are probably taken for granted in a lot of cases. While it is a significantly easier problem to fix than incorrect symmetry or mass models, it is still an assumption which can change your end value for mass. We re-centered the annular regions five times, the first four of which were 9 arcseconds or 20 kiloparsecs (1 parsec = 3×10^{16} km) from the correct centering and the last was about 26 arcseconds or 50 kiloparsecs from the correct centering. For a table with the final hydrostatic mass value of all of these miscenterings and how they compare to the nominal hydrostatic mass, see Table 5. The first miscentering was to the northwest, and resulted in a final hydrostatic mass of around 2×10^{14} solar masses. Compared to the correct centering, which had a final mass of 1.3×10^{15} solar masses, this is 6.5 times smaller. The second was to the southwest and this miscentering resulted in a hydrostatic mass around 1.2×10^{15} solar masses. This mass is very close to that of the correct centering, but the two are distinguishable through comparison of the graphs of their hydrostatic mass versus radius, see Figures 4 and 5. The third miscentering was to the southeast, and resulted in a hydrostatic mass of 10^{18} solar masses. This mass is much too high for a galaxy cluster, as the most massive galaxy clusters are around 10^{16} solar masses. The fourth miscentering was to the northeast and resulted in a hydrostatic mass of around 1.1×10^{15}, again very close to that of the correct centering. The fifth and final miscentering which was placed significantly further from the correct centering than the others resulted in a hydrostatic mass of around 3×10^{15}, roughly twice the mass of the correct centering, and its hydrostatic mass profile can be seen in Figure 6. Ignoring the 10^{18} solar mass outlier, the standard deviation of these miscenterings was calculated to be 1.016×10^{15} solar masses, and the standard error to be 0.5079×10^{15} solar masses. This corresponds to an overall 39% mass error from these miscenterings. Therefore, it is safe to say physicists should not assume their regions are correctly centered if they desire accurate mass estimates.

Attempt	$M(< r)$ $(10^{15}\ M_\odot)$	$\frac{M - M_n}{M_n}$
Nominal	1.3	0
9 arcsec NW	0.2	-0.84615
9 arcsec SW	1.2	-0.076923
9 arcsec SE	1,000	768.23
9 arcsec NE	1.1	-0.15385
26 arcsec	3	1.3077

Table 5 – The total enclosed mass of each miscentering of Abell 2029 accompanied by a relative error (where M_n is the nominal mass).

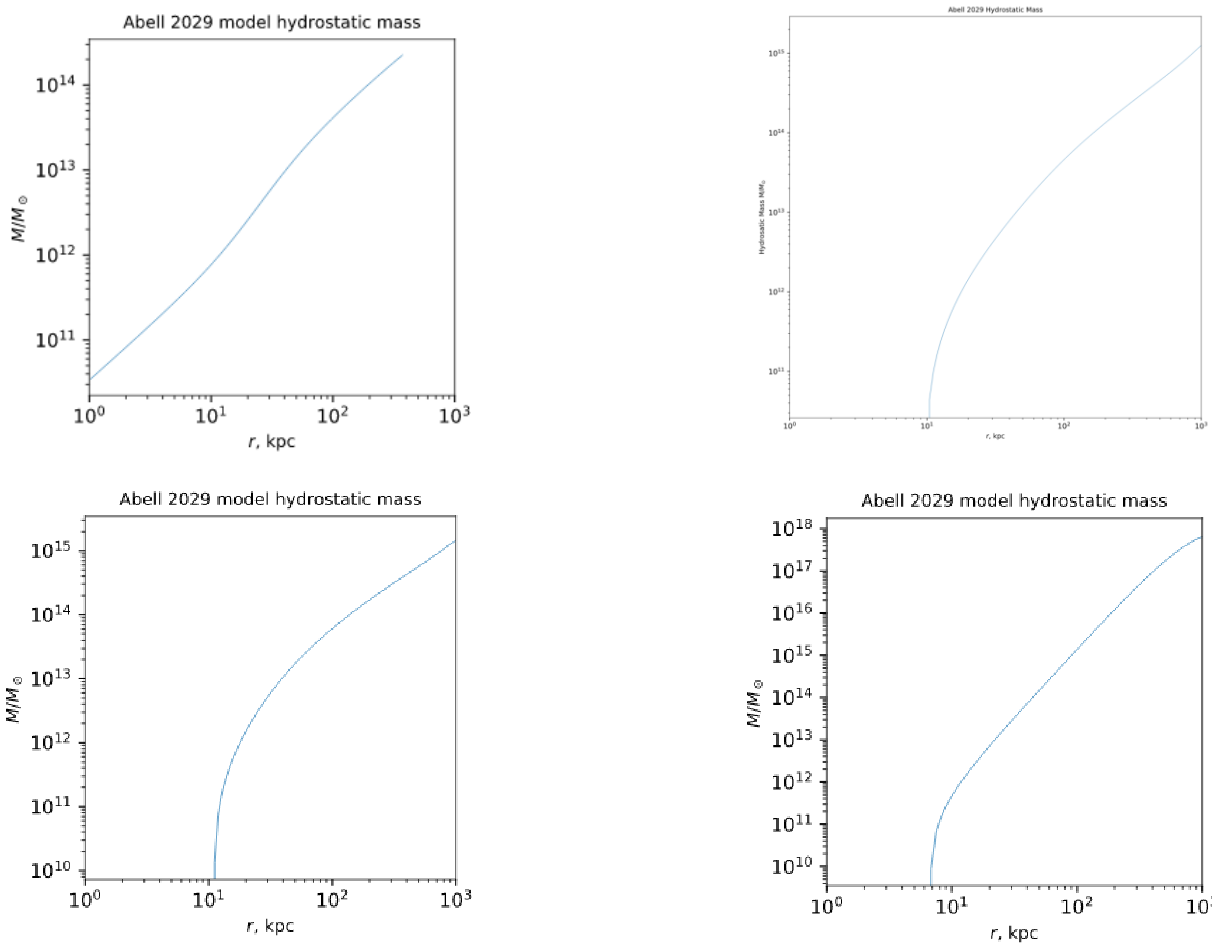

Figure 5 – The mass profiles of the various 9 arcsec (20 kpc) miscenterings of Abell 2029. The top left profile is that of the northwest centering, the top right that of the northeast, the bottom left that of the southwest, and the bottom right that of the southeast.

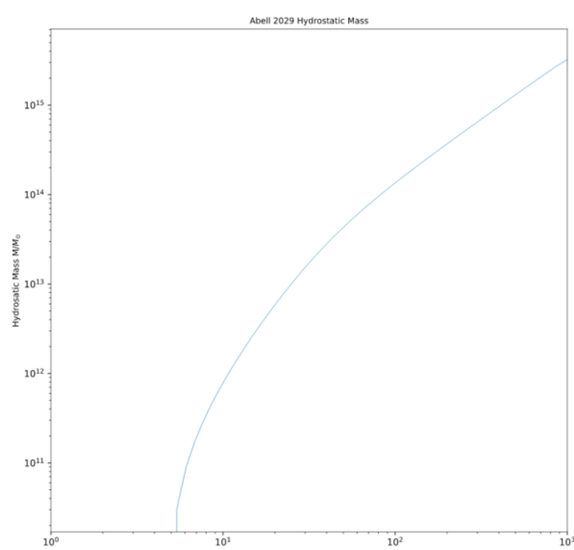

Figure 6 – The mass profile of the 26 arcsec (50 kpc) miscentering of Abell 2029.

3.3 – Calibration

We were able to experiment with the uncertainty stemming from calibration by varying the n_H value while fitting the best-fit temperature model to Abell 2029 in XSEPC. The n_H value was raised once and lowered once so that the overall temperature was roughly 10% higher or lower. After deriving the hydrostatic mass with these new temperatures, a final mass of 1.13×10^{15} was found after raising n_H and a final mass of 1.85×10^{15} was found after lowering n_H. This corresponds to a 13% decrease and a 42% increase of the original mass value, respectively. The resulting graphs can be found in Figure 7.

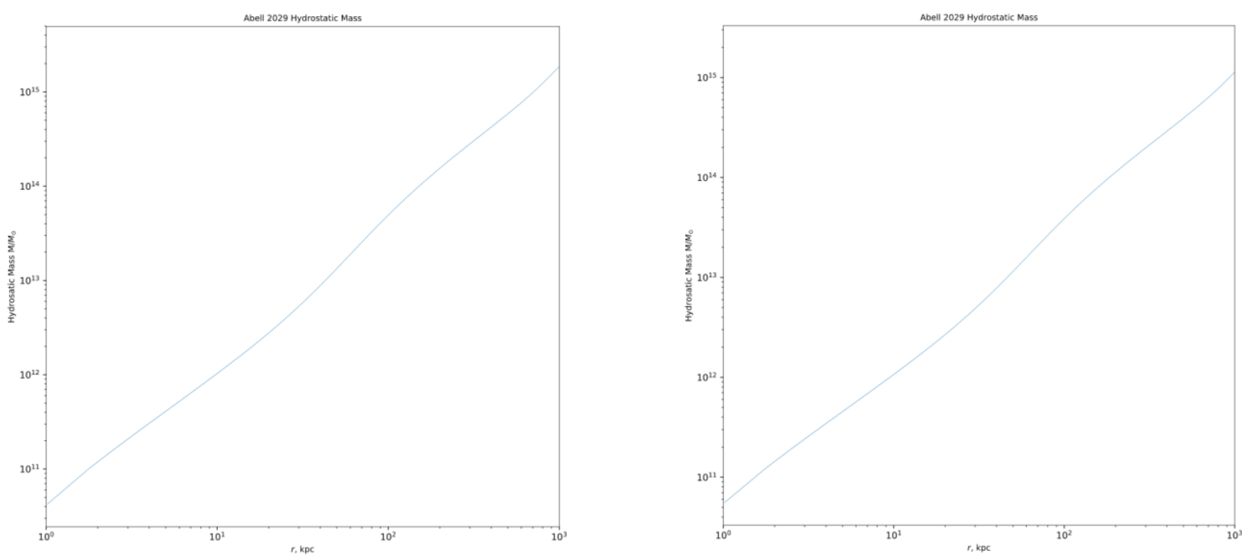

Figure 7 – The mass profile of Abell 2029 after lowering n_H (left) and raising n_H (right).

3.4 - Hydrostatic Equilibrium and Symmetry

There exist certain assumptions that are much less subtle in how they will change the overall mass estimate of a galaxy cluster. For those wanting to use the hot X-ray-emitting gas inside of galaxy clusters in order to derive mass estimates, as is the case for this research project, there are assumptions that can be made in order to greatly simplify the derivation, as previously mentioned. Some of these assumptions will lead to obvious errors in the final mass estimate and would result in no better restraints for the cosmological parameters mentioned before. One such assumption is that the gas in the galaxy cluster is in hydrostatic equilibrium. Assuming we are working with gas in hydrostatic equilibrium allows us to use a rather simple formula for finding the hydrostatic mass, but this equation also assumes spherical symmetry. Spherical symmetry is the other assumption that will lead to glaring errors in the final mass estimate, as no galaxy cluster is naturally perfectly spherical. Therefore, assuming spherical symmetry will always lead to some form of error. To see this, we derived Abell 2029's mass once more using the corrected hydrostatic mass equation (equation 4) and plotted the results in Figure 8.

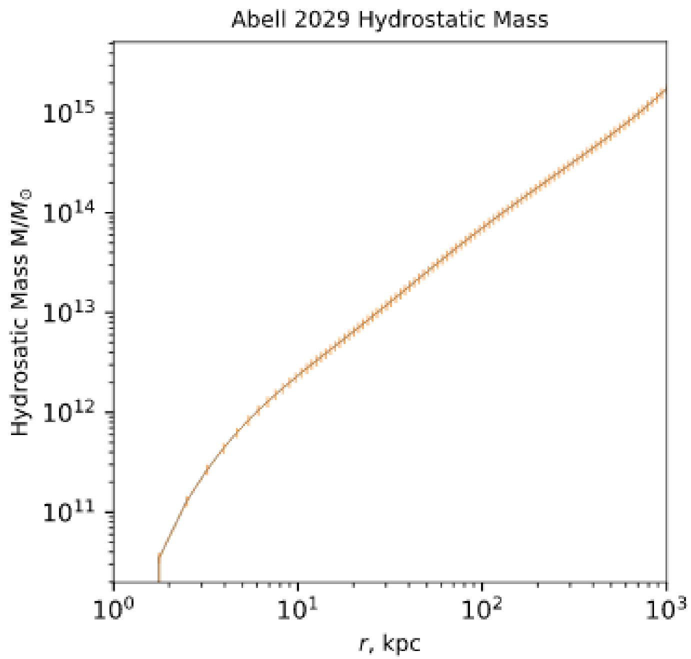

Figure 8 – The corrected mass profile of Abell 2029.

The resulting final mass value of the corrected hydrostatic mass was 1.75×10^{15}, roughly 1.35 times larger than the original mass. As you can see, using this X-ray method for estimating the mass of a galaxy cluster clearly has its disadvantages that manifest in the form of errors in the final mass estimate when these assumptions are used. The X-ray method does have advantages over various other methods of mass estimation, however, with one big advantage being that the mass estimate is derived without needing to know anything about the dark matter distribution.

DISCUSSION

4.1 – Calibration, Galaxy Cluster Evolution and Variable Relations

There have been somewhat similar research projects to this one that have tested various different assumptions and uncertainties. One such project was performed in a series of papers by Vikhlinin et al. in 2008 titled “Chandra Cluster Cosmology Project”
and in this project X-ray data from the Chandra telescope was used in order to identify certain systematic errors in galaxy cluster mass estimates. The main sources for these systematic errors were separated into three groups. The first group was the uncertainty in the calibration of Chandra hydrostatic mass estimates. The second group consisted of various uncertainties related to the evolution of galaxy clusters, an example of which being departures from the expected relation between galaxy cluster mass and the cluster’s average temperature. The last group was uncertainties from the evolution of the X-ray luminosity and galaxy cluster mass relation. To determine the uncertainty stemming from the calibration of Chandra hydrostatic mass estimates, Vikhlinin et al. compared masses obtained from Chandra data to masses obtained from weak lensing studies. The resulting value for this systematic uncertainty was 9%, which is high enough to be of concern for those seeking accurate values for cosmological parameters, but not quite as high as the error derived in this project. The uncertainty in the evolution of galaxy clusters, specifically in the overall mass and average temperature, as well as the overall mass and gas mass relations, was calculated by finding the amount of corrections that had to be applied to numerical models of galaxy cluster formation. The resulting systematic uncertainty was between 5% and 6%, again large enough to warrant being accounted. The uncertainty from the X-ray luminosity and galaxy cluster mass relation was found to be negligible for the high end of the galaxy cluster mass function, but the scatter of this relation was calculated to be around 10%.

This is obviously a very large scatter, and such a scatter would cause uncertainties in any formula or derivation that required an accurate estimation of this relation.

4.2 – Redshift

Another similar project comes from Caltech (Weinberg, D. H., et al 2013) which focused on galaxy clusters in general, but had a section about systematic uncertainties. The section gives a formula for calculating the expected number of galaxy clusters of a certain volume V at some redshift Z. The Caltech project claims that X-ray spectroscopy, a popular and reliable way to measure redshift, has an error of about 0.03 when observing the iron lines. This mainly effects high redshift clusters, as for such clusters this form of redshift determination is the only one we have and thus we cannot compare values to find the true value for redshift. Clusters at smaller redshifts are closer to us and therefore we can get more infrared information from their spectra, making it easier to determine the redshift accurately. This redshift inaccuracy at higher redshifts could drastically affect the outcome of the mass derivation process for a galaxy cluster, as interpreting the spectrum for the galaxy cluster relies on knowing its redshift. Producing images is indirectly affected by this, as having the wrong redshift means using the wrong physical conversion due to a potentially incorrect angular scale and physical distance. Said images are of great importance for determining the brightness and mass of galaxy clusters, as they are where the data from the annular regions are coming from.

4.3 – Dark Matter Halo Mass Function

The Caltech project goes on to show another form of systematic uncertainty which manifests as the error in dn/dM, which is the halo mass function. Every galaxy cluster has a “halo” of dark matter surrounding it and keeping it gravitationally bound. This halo mass function measures the number density of dark matter halos for each mass interval. Caltech’s formula for detecting the number of galaxy clusters within a certain volume depends on this halo mass function and thus requires knowing what the function is in order to use the formula. If the uncertainty in this mass function were to exceed the error in the number of clusters, then the end result would be limited due to theoretical uncertainty rather than observational errors. The paper shows that a 2008 study from Tinker et al. capped the uncertainty of the halo mass function at 5%, meaning the number of clusters had to have a fairly low error. If the halo mass function had a decent amount of error, then the computation of the amount of galaxy clusters of a certain mass would end in an incorrect result, as would the mass estimates of said galaxy clusters due to an incorrect value for the density of dark matter halos.

4.4 – Hydrostatic Equilibrium and Symmetry

A paper by R. Sadat titled "Clusters of Galaxies and Mass Estimates," which talks about the various methods one can use to estimate the mass of a galaxy cluster, has some interesting results in just how inaccurate this X-ray method of galaxy cluster mass estimation can be. The paper goes over the results of research conducted by Balland and Blanchard in 1995, which found that the resulting accuracy of these mass estimates assuming hydrostatic equilibrium was much poorer than had been initially claimed. To test this, they applied this mass estimate procedure to the Coma cluster, and they ended up finding that there was at least a factor of 2 uncertainty in the mass. Needless to say, this is much too high of an uncertainty for the purpose of restraining cosmological parameters. Additionally, the paper goes over the errors stemming from the assumption of spherical symmetry. These errors come from the fact that galaxy clusters have very complex morphologies due to underlying substructures. The paper goes over how various numerical simulations have shown that the masses of galaxy clusters which are merging with other universal structures (like other galaxy clusters) are usually underestimated. This comes from the fact that some of the energy of the galaxy cluster's gas would be in the form of kinetic energy rather than just in thermal energy. This means that there is less thermal pressure support needed to remain in hydrostatic equilibrium and since pressure is directly proportional to temperature due to the ideal gas law, the temperature will be lower and thus the mass will be underestimated. The paper shows that this underestimation can be as large as 40%, as found by Schindler in 1996. This number speaks for itself, as underestimating the masses of clusters by this much will in no way help in restraining cosmological parameters, and using data like this may lead to completely incorrect values for such parameters.

In 1997, Balland and Blanchard published a paper titled “On the Uncertainty in X-Ray Cluster Mass Estimates from the Equation of Hydrostatic Equilibrium” which completely focused on the uncertainties that come from assuming hydrostatic equilibrium when deriving mass estimates. They used two particular clusters as examples: the Perseus cluster and the Coma cluster. Balland and Blanchard found that the Perseus cluster had an initial uncertainty factor of 3 after deriving its mass while assuming hydrostatic equilibrium, with the uncertainty factor being larger than 10 at a large radius. They did manage to lower this uncertainty to around a factor of 1.5 and 3 at large radius through placing certain logical conditions on the cluster, such as assuming the temperature is nonzero up to where significant X-ray emission is found. Balland and Blanchard commented that this substantial uncertainty might have come from data of poor quality. The results of the Coma cluster were mentioned previously, and the Coma cluster ended up having an uncertainty factor of 2, even with better quality data. Therefore, Balland and Blanchard came to the conclusion that cluster masses cannot be determined accurately when using the equation of hydrostatic equilibrium even if excellent quality X-ray data is used.

CONCLUSION

It is quite clear that all of the systematic errors listed should be considered when deriving an accurate value for a galaxy cluster's mass. The most substantial of these errors is unarguably the assumptions of hydrostatic equilibrium/spherical symmetry and calibration. The uncertainties coming from galaxy cluster evolution, variable relations, redshift and the dark matter halo mass function were for the most part high enough to warrant attention, but they were not directly tested in this paper and were ultimately not the focus of this project. An uncertainty that was a focus was the centering of your regions around your cluster and, depending on how close your regions are to the correct center of your cluster, your end mass could be overestimated by up to 130% or underestimated by up to 85%. This does include a region whose center was 50 kpc away from the correct centering which is so far away from the actual center that this is very unlikely to happen by accident. Thus, one should really only consider the regions with an 85%, 8% and 15% decrease in the mass estimate. While all of these are high enough to be concerning for someone looking for a very good estimate for a cluster's mass, they depend on the professional carrying out the experiment more than anything else. Although this uncertainty can be avoided by someone with enough experience, it still creates enough of a mass difference to force physicists to double-check the centers of their regions in the future.

The results from this project show us that calibration plays a large role in the uncertainty of galaxy cluster mass estimates. Unfortunately, calibration isn't something that can be solved so easily like the correct centering of your regions can be. Telescopes like XMM and Chandra have their own unique calibrations, and no one is sure which telescope is correct. From this project, we see through varying n_H in XSPEC that calibration can have an impact as big as a 42% increase in the mass estimate. This is obviously a large overestimate of the cluster's mass, which would lead to incorrect values of cosmological parameters. Another way to see how big of an affect calibration has on the final mass estimate of a cluster is to compare the mass retrieved from a telescope such as Chandra to the mass retrieved from another method, such as another telescope or gravitational lensing. In the series of papers titled "Chandra Cluster Cosmology Project," Vikhlinin, A, et al. performed this by comparing mass estimates form Chandra to those from weak gravitational lensing. The resulting value for the systematic uncertainty was 9% which, while lower than the uncertainties found for the calibration from this project, is still substantial enough to need to be accounted for in order to better restrain cosmological parameters.

From the outcomes of this study, we see that after correcting for the non-thermal pressure of Abell 2029's gas, the mass estimate increases by about 35% compared to the non-corrected mass. Since hydrostatic equilibrium assumes all pressure coming from the gas is thermal in origin, this shows that assuming hydrostatic equilibrium will most likely result in an underestimate for the mass of your cluster, as it is unlikely a galaxy cluster is completely in hydrostatic equilibrium. Additionally, we see from Balland and Blanchard's work that it does not matter the quality of data you use for your mass estimation; it will be wrong if you assume hydrostatic equilibrium. This, in addition to the results of this project, lead me to believe that the assumption of hydrostatic equilibrium is the biggest source of error for galaxy cluster mass estimates. Since calibration heavily depends on the instrument you are using, you may not need to account for it as often as you need to account for non-thermal pressure in a galaxy cluster. Also, the amount of error coming from the centering of regions is really up to the person creating the regions, so this also does not have to be accounted for every time. The assumption of hydrostatic equilibrium is almost never going to be completely correct, so this source of error is one that will need to be accounted for nearly every time you estimate the mass of a galaxy cluster. Since restraining cosmological parameters requires our mass estimations to be as accurate as possible, we must keep in mind this non-thermal pressure correction for these masses so that they are much more accurate in the future.

While symmetry was not explicitly tested in this paper, it was indirectly tested through the testing of the assumption of hydrostatic equilibrium. We saw just how big of an impact the assumption of hydrostatic equilibrium has on the mass estimate of a cluster, and since hydrostatic equilibrium assumes spherical symmetry this means that assuming spherical symmetry should also have a rather large impact. Due to time constraints, this symmetry assumption could not be tested in depth. Future projects, or further continuations to this project, should investigate this avenue of additional uncertainty. Instead of spherical symmetry, it might be more appropriate to consider elliptical symmetry, as a good number of clusters are elliptical rather than spherical. Once the underlying assumptions about symmetry are changed, the mass could be derived once more with different kinds of symmetry to see which gives the most accurate value for mass. This will also reveal which kinds of symmetry give the most uncertainty in the mass. Finally, once we know how much uncertainty in the mass estimate comes from spherical symmetry, in addition to what we already found out about how much uncertainty comes from assuming hydrostatic equilibrium and from calibration, we will be better informed on how to create more accurate galaxy cluster mass estimates in the future. With these more accurate mass estimates we will be able to place better constraints on cosmological parameters and from this hopefully answer some of the many questions we have about the universe.

APPENDIX

1. The density parameter represents the ratio of the average density of the universe to the critical density. The density parameter is the sum of all densities of the universe ($\Omega_0 = \Omega_m + \Omega_r + \Omega_\Lambda$) like matter ($\Omega_m$), radiation ($\Omega_r$) and dark energy ($\Omega_\Lambda$), where dark energy is the hypothesized energy that accelerates universal expansion. The critical density is the density the universe would have to be if it were spatially flat, and thus the universe would be deemed flat if $\Omega_0 = 1$. The true value of Ω_0 has been found to be close to 1, meaning we live in an almost completely flat (or perhaps even completely flat) universe. Our universe being flat would imply that it would have enough matter to stop expansion after an infinite amount of time. If Ω_0 is greater than 1, the universe would eventually stop expanding and collapse in on itself. If Ω_0 is less than 1, the universe would expand for all eternity.
2. Quintessence claims that dark energy has an equation of state value different from -1 and is actually a fifth fundamental force[7] (with the current four fundamental forces including the weak force, the strong force, electromagnetism and gravity). Quintessence also claims that dark energy changes over time[8], which directly opposes the generally accepted cosmological constant idea of dark energy that says dark energy has been the same since the Big Bang.

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