University of Utah
UNDERGRADUATE RESEARCH JOURNAL

PERSISTENT CURRENT FROM SPIN ORBIT INTERACTION IN HELICOIDAL QUANTUM WIRES

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We study, theoretically, the system of a helix shaped quantum wire with spin orbit coupling (SOC). We say the system has conducting electrons. We find that in a certain regime, the coupling direction changes due to the interaction between the curved path of the electrons and the spin orbit coupling. The coupling direction changes from being along the long axis of the wire to perpendicular to it. When this happens, we say the wire is in resonance. The spin orbit coupling leads to a band structure with two intersecting bands. At resonance, a parallel magnetic field can open a gap between the two bands, lowering the energy of the system by $\Delta U_B \propto B^2 \ln \frac{c}{B}$. But a current in the wire will generate such a field but raises the energy of the system by $\Delta U_I \propto B^2$. When these contributions are put together, the minimal-energy configuration (the one the system will be in at equilibrium) has a non-zero current, as well as a non-zero spin current, thus we predict a dissipationless current will spontaneously arise in the system.

To state our predictions for the current, we use a unitless value related to the band structure shift of the wire as a result of this effect:

$$\Delta J = \pm \frac{\sin\alpha}{\exp 1/2} \chi_J \exp[-\frac{3}{4}\cos^2\alpha\, \chi_J^2]$$

$$\chi_J = \frac{4\pi\hbar \sin\alpha}{R\mu_B\mu_0 e\cos^2\alpha}$$

There will be a net current of:

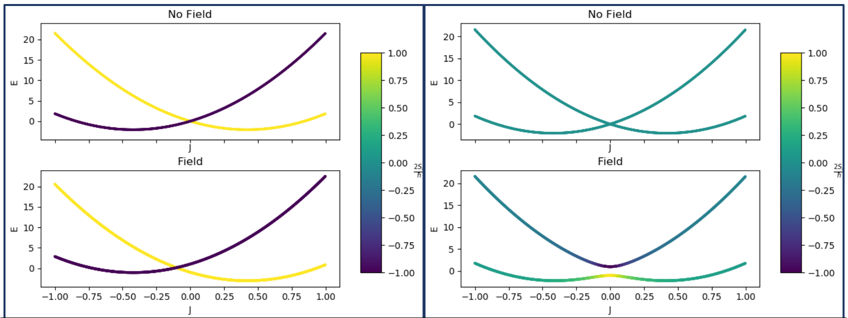
$$I = \frac{2\mathrm{e}\lambda\cos\alpha\sin\alpha}{\mathrm{R}}\Delta J$$

And a spin current of:

$$S = \frac{\mu_B\lambda M_u \sin\alpha\cos\alpha}{R}\frac{\Delta J^2}{\sqrt{\left(\frac{\lambda\sin\alpha}{R}\right)^2 + (\mu_B M_u \Delta J)^2}}$$

$$M_u = \frac{\mu_0 e\cos^2\alpha}{2\pi R^2\hbar}$$

Notice that the charge (electrical) current can reach equilibrium in either the positive or negative direction (from the $\pm$ sign in ΔJ) but the spin current can only go in one direction (because it has a term ΔJ^2, which is always positive). This system has both spontaneous breaking of one symmetry (time-reversal) and a fundamental lack of another symmetry (inversion) so that the spin current will take on a positive non-zero value under the right conditions.



Plot of the band structure from SOC but without interactions out of resonance (left) and in resonance (right). The colormap represents the average spin of the energy eigenstates. In resonance, without a magnetic field along the wire's long axis, the eigenstates have no average spin in any direction because the spin direction varies with position and is wound around the wire, so it averages out to zero.

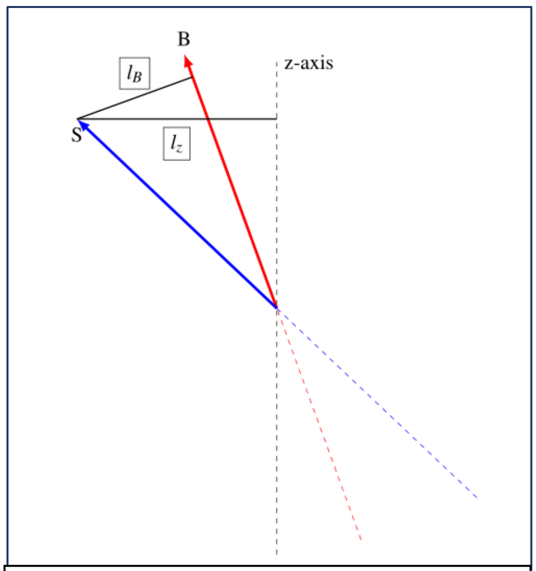
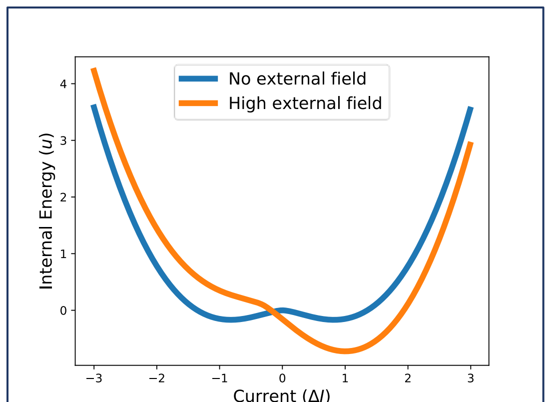


Diagram showing the direction of spin-energy splitting due to SOC. The relative magnitude of the marked lengths corresponds to parameters of the wire. When the frequency of movement around the central axis of the wire is similar to the frequency of spin precession due to the orbit induced magnetic field, the spin direction pulls away from the induced field, which is near the z-direction and at some point is pointed horizontally (around the cylinder of the helix).



Plot showing the internal energy of the system at zero temperature versus the current in the system. The axis are linear but the scale is arbitrary. Notice the two minima in the absence of an applied external magnetic field.