Security Design in Markets with Risk: Price and Allocational Efficiencies

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Research question

This research uses an experimental approach to examine the impact of security design (defined as correlation between assets in the market) on the ability of markets to achieve allocational efficiency. This will be done by manipulating the payoffs of the assets available in the market to induce positive, negative, or zero correlation between them and seeing how trading outcomes change as a result.

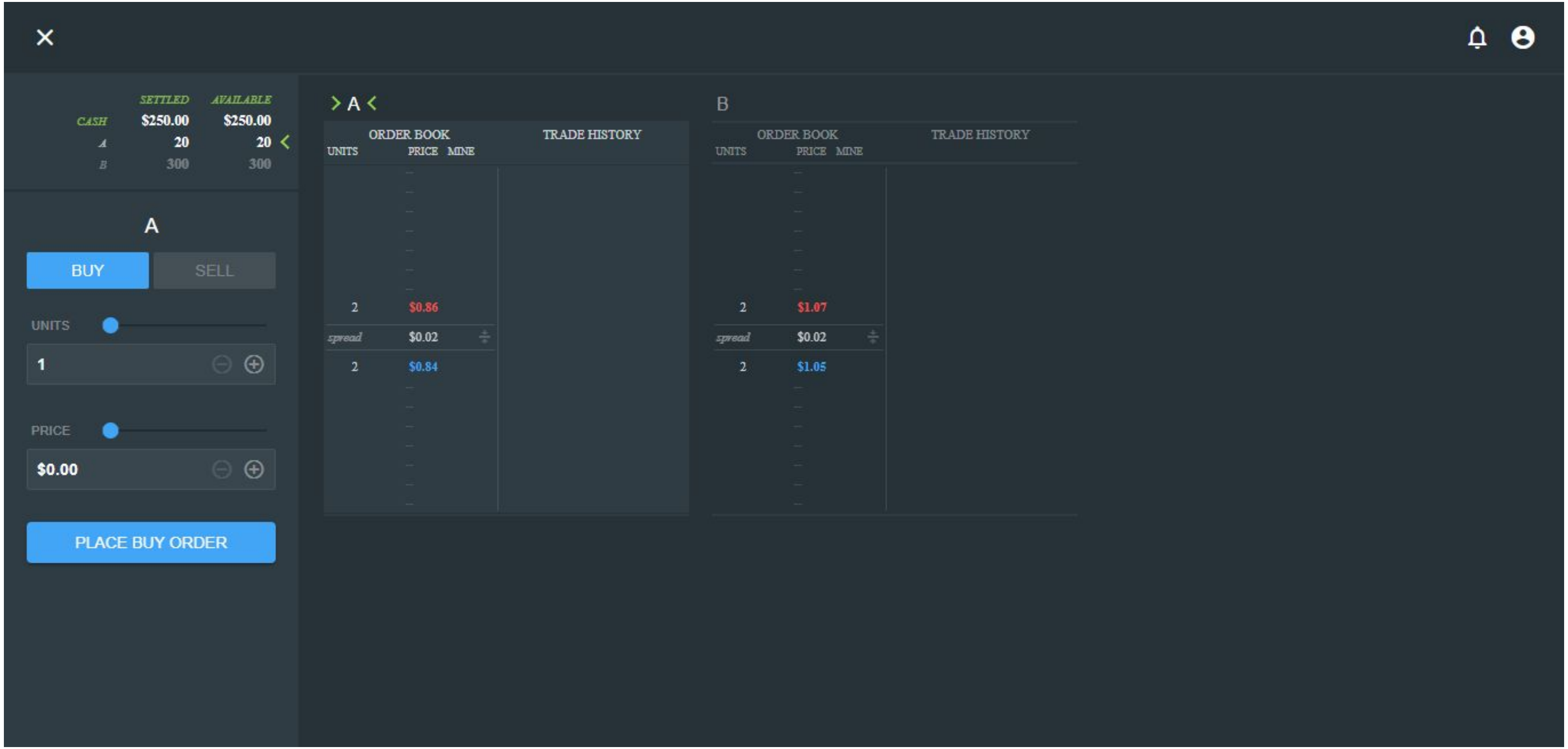
Motivation

- Increasing popularity of index funds which tend to be positively correlated
  - Our hypothesis is that market participants have a harder time reaching their optimal consumption holdings when the assets in the market are positively correlated
  - Bogle (2016)
- Why experimental finance?
  - Importance of testing theory
  - Researchers have more control over market fundamentals

Finance experiments

- Human subjects sit at computer terminals and use software (flexemarkets for this research) to trade with each other
- Researchers are interested in watching what happens to resulting prices and holdings of the assets when they change certain market fundamentals
    - Security design is the fundamental of interest for this research

Flex-e-Markets



Relevant literature

- Experimental evidence related to ability of markets to achieve allocational efficiency:
  - Asparouhova, Bossaerts, and Ledyard (2019)
  - Bossaerts, Plott, and Zame (2007)
- Asparouhova, Bossaerts, and Ledyard (2019) produce initial evidence suggesting that markets with negatively correlated assets are faster to reach allocationally efficient outcomes

Security design

- Operationally defined as the correlation between the risky assets in the market
- Informally: correlation measures how closely two variables move together
- Formally: Correlation (ρ) is a scaled version of covariance that is bounded between -1 and 1 where ρ=-1 represents perfect negative correlation and ρ=1 represents perfect positive correlation

Variance(A)=$\boldsymbol{\sigma}_A{}^2$=($\Sigma$($A_i$-E[A])/(n-1)

Standard deviation(A)=$\boldsymbol{\sigma}_A$=($\boldsymbol{\sigma}_A{}^2)^{1/2}$

Cov(A,B)=$\boldsymbol{\sigma}_{A,B}$=($\Sigma$($A_i$-E[A])($B_i$-E[B])/(n-1)

Correlation(A,B)=$\rho_{A,B}$=$\boldsymbol{\sigma}_{A,B}$/($\boldsymbol{\sigma}_A$*$\boldsymbol{\sigma}_B$)

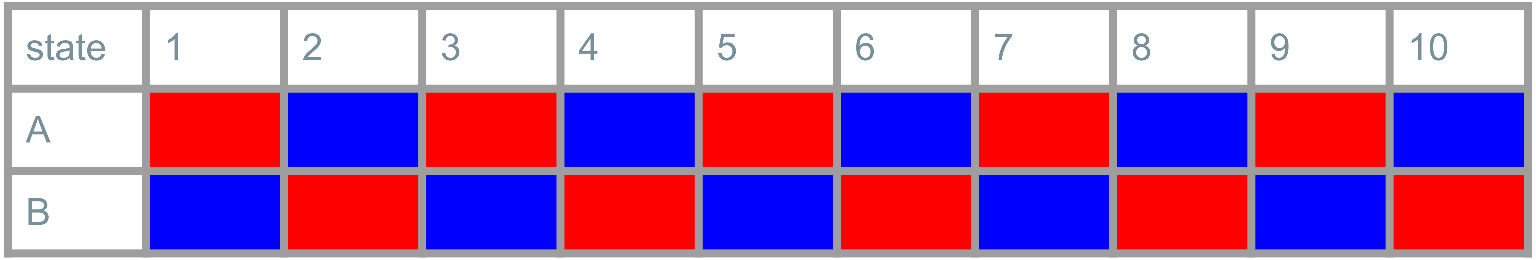
Correlation illustrated

Positive correlation:

state	1	2	3	4	5	6	7	8	9	10
A										
B										

Correlation illustrated

Negative correlation:



Experimental design

- 2 risky assets and 1 riskless asset (cash)
- Risky assets have uncertain payoffs between 0 and 2
- Cash always pays out 1
- Experiments will consist of three possible "treatments":
  1. Market where two risky assets are negatively correlated ($\rho<0$)
  2. Market where two risky assets are uncorrelated ($\rho=0$)
  3. Market where two risky assets are positively correlated ($\rho>0$)

Negative correlation treatment

State	boom	recession	normal
Asset A	2	0	1
Asset B	0.5	1	1.5
Cash	1	1	1

$$\rho_{A,B} = -0.5$$

Zero correlation treatment

State	boom	normal	recession
Asset A	2	0	1
Asset B	1.5	1.5	0
Cash	1	1	1

$$\rho_{A,B} = 0$$

Positive correlation treatment

State	boom	recession	normal
Asset A	2	0	1
Asset B	1	0.5	1.5
Cash	1	1	1

$$\rho_{A,B} = 0.5$$

Timeline of research project

- 2 main phases of the project
    1. Simulations using trading robots (make trades according to a specified algorithm)
    2. Experiments with human subjects making trades manually
        a. Cancelled due to coronavirus

Simulations with trading robots

- We call the robots used in these simulations “quadratic utility maximizing” robots
    - In economics, we use utility functions to make outcomes of models, experiments, analysis, etc. more quantifiable
- For the simulations, we assume that individuals have quadratic utility functions:

$$U_i(x_i) = x_i - b_i*(x_i)^2$$

Risk aversion parameter b_i

- In the utility function $U_i(x) = x_i - b_i * (x_i)^2$, b_i is known as a *risk aversion parameter*
  - The risk aversion parameter is a constant that determines how much an individual is penalized for having potential payoffs that are different in different states
- Example: choose between receiving $50 for sure or a coin flip where heads means receiving $100 and tails means receiving $0

Expected value of a quadratic utility function

- Expected utility:

$$EU^i = \pi_1(x^i_1 - b^{i} * (x^i_1)^2) + \pi_2(x^i_2 - b^{i} * (x^i_2)^2) + \pi_3(x^i_3 - b^{i} * (x^i_3)^2),\ i=1,2$$

- π_i represents probability
- This utility is in terms of income **x**
    - in order for a robot to use it for making trades it needs to be in terms of available market securities

Expected utility functions in terms of holdings

- We need an expected utility function in terms of available market securities
- First, define current holdings vectors of the 3 available market securities:

  $\mathbf{h}^i = (a^i, b^i, c^i)$, $i=1, 2$ (meaning individual i currently holds a^i units of risky asset A, etc.)

- As well as vectors describing payoffs of risky assets A and B in each of the three states:

  $\mathbf{A} = (a_1, a_2, a_3)$
  $\mathbf{B} = (b_1, b_2, b_3)$

Expected utility in terms of holdings (continued)

- Expected utility in terms of market securities:

$$
\begin{aligned}
EU^i = {} & \tfrac{1}{3}\left(a^i a_1 + b^i b_1 + c^i - B^i(a^i a_1 + b^i b_1 + c^i)^2\right) \\
& + \tfrac{1}{3}\left(a^i a_2 + b^i b_2 + c^i - B^i(a^i a_2 + b^i b_2 + c^i)^2\right) \\
& + \tfrac{1}{3}\left(a^i a_3 + b^i b_3 + c^i - B^i(a^i a_3 + b^i b_3 + c^i)^2\right), \qquad i=1,2
\end{aligned}
$$

Marginal valuation calculations

- Robots take partial derivatives of the expected utility function to determine how much each asset is worth to an individual with specified holdings:

$(\partial U/\partial a^i) = E[\mathbf{A}](1 - 2B^i c^i) - \frac{2}{3}B^i(a^i((a_1)^2 + (a_2)^2 + (a_3)^2) + b^i(a_1 b_1 + a_2 b_2 + a_3 b_3))$

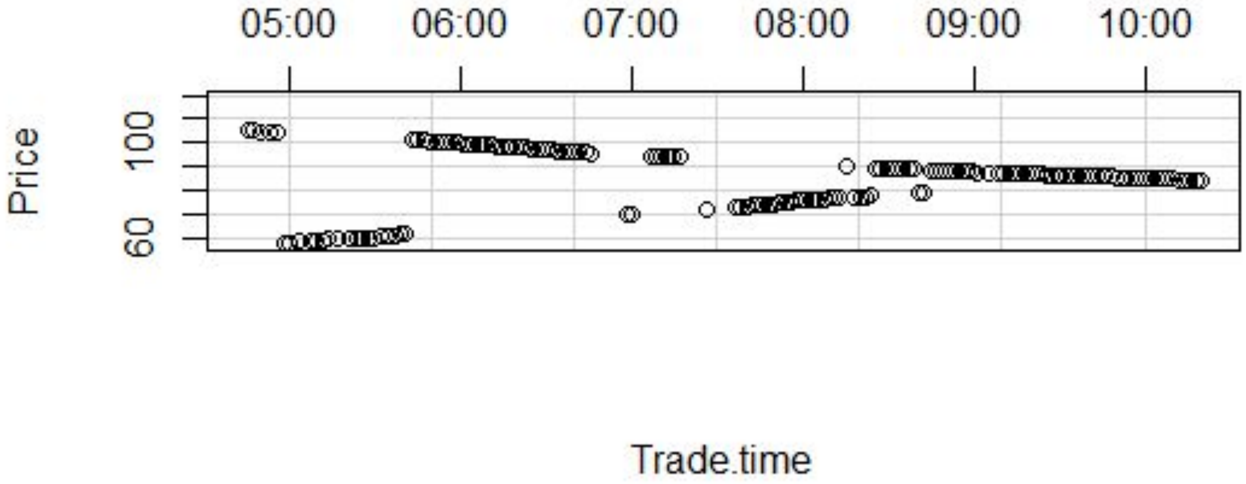
$(\partial U/\partial b^i) = E[\mathbf{B}](1 - 2B^i c^i) - \frac{2}{3}B^i(b^i((b_1)^2 + (b_2)^2 + (b_3)^2) + a^i(a_1 b_1 + a_2 b_2 + a_3 b_3))$

$(\partial U/\partial c^i) = 1 - 2B^i a^i E[\mathbf{A}] - 2B^i b^i E[\mathbf{B}] - 2B^i c^i$

Marginal valuation calculations (continued)

- $\partial U/\partial a^i$, $\partial U/\partial b^i$, and $\partial U/\partial c^i$ are the marginal valuations of the risky assets A and B and the riskless asset cash (respectively)
    - Once the robot calculates this marginal valuation, it attempts to maximize an individual's utility by posting a buy order below the valuation and a sell order above the valuation
    - How far above and below is determined by a spread parameter that the robot takes as input

Robots trading with each other - example



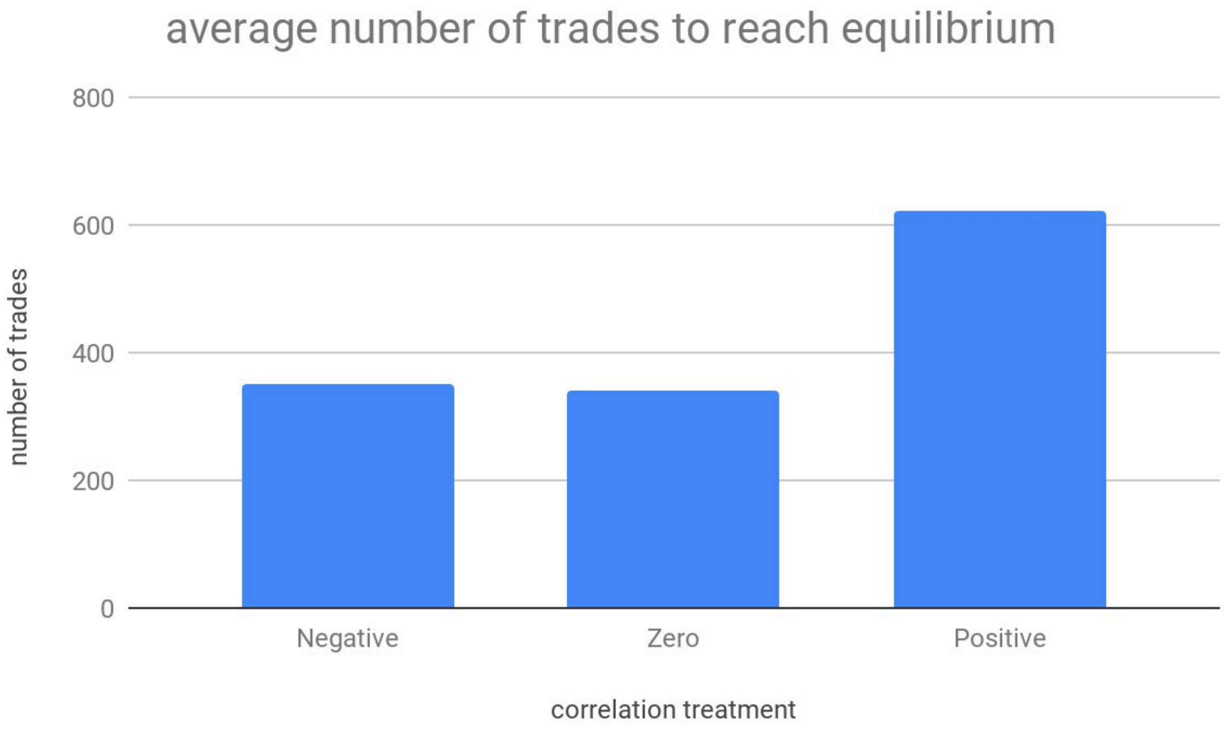
Measuring allocational efficiency

- Allocational efficiency: how effectively are markets able to make participants better off i.e. increase their utility
- Simplest way: how many trades does it take for the market to reach equilibrium?
- In the context of this research, equilibrium occurs when the two individuals have equal marginal valuations for both of the risky assets

Allocational efficiency measured by number of trades

- Using this logic, we can say that a market that takes more trades to reach Pareto optimality is less allocationally efficient than one that takes less trades
  - The path to efficiency is longer

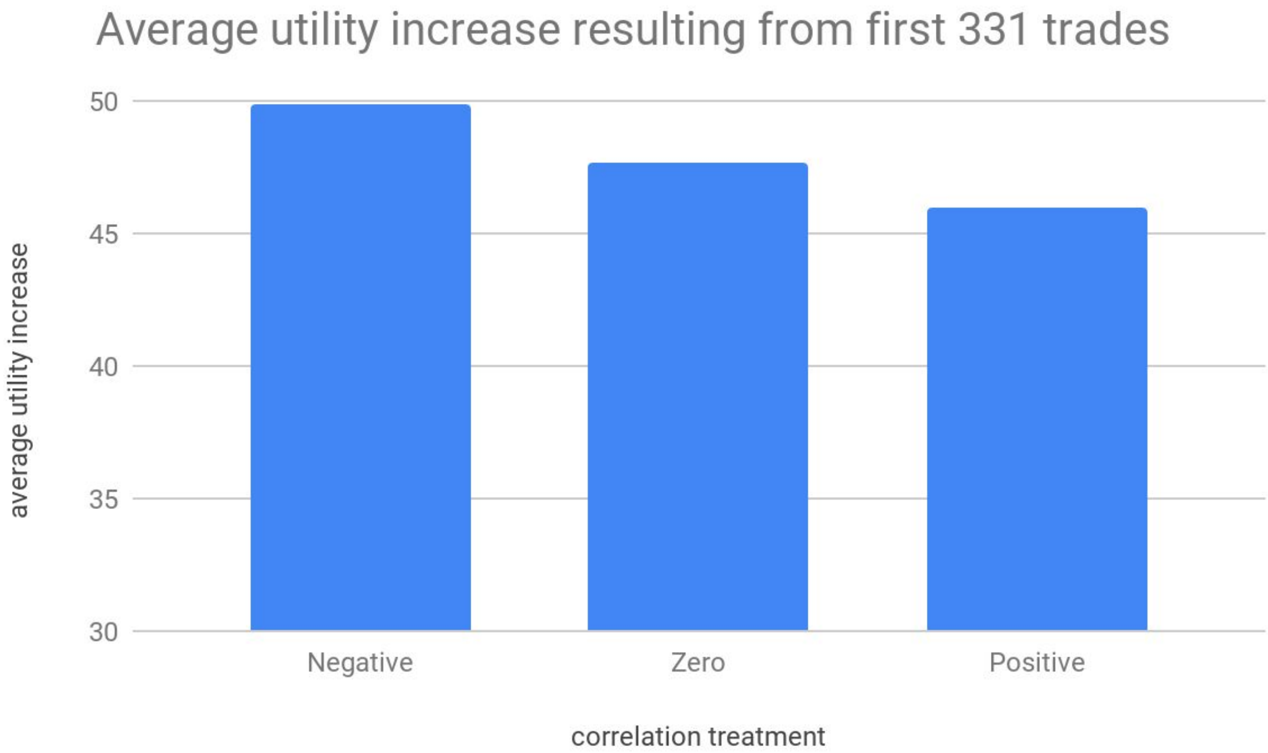
Initial simulation results



A better way to measure allocational efficiency

- Look at change in utility resulting from trades rather than only number of trades
- Methodology:
  - Determine a cutoff number of trades to apply to each simulation and measure how much utility has improved as a result of those trades
- We choose the cutoff to be the smallest number of trades it took out of all simulations to reach equilibrium
  - Zero correlation market simulation #2 → 331 trades to equilibrium

Results from second method



References

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